On the algebraic approach to quantum Dirac fields

Simone Murro

Department of Mathematics
University of Regensburg

COLOQUIO DE MATEMÁTICA UC

Santiago de Chile, 4th of November 2016

Motivations and goals of my talk

" *What is a* **QFT?** " $\xrightarrow{\text{for a deeper understanding}}$ **mathematical axioms** for **QFT**

- **Algebraic QFT** :
  - ✓ $\mathcal{M} \supset \mathcal{O} \mapsto \mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\mathcal{M}) := \bigcup_{\mathcal{O}} \mathcal{A}(\mathcal{O})$
    - **Isotony:** if $\mathcal{O} \subseteq O' \Longrightarrow \mathcal{A}(\mathcal{O}) \subseteq \mathcal{A}(\mathcal{O}')$
    - **Causality:** if $\mathcal{O} \cap J(\mathcal{O}') = \emptyset \Longrightarrow [\mathcal{A}(\mathcal{O}), \mathcal{A}(\mathcal{O}')] = 0$
    - **Covariance:** isometry ι of M, $\Longrightarrow \alpha_\iota \in \mathrm{Aut}(\mathcal{A})$ s.t. $\alpha_\iota \mathcal{A}(\mathcal{O}) = \mathcal{A}(\iota(\mathcal{O}))$
  - ✓ $\omega : \mathcal{A}(\mathcal{M}) \to \mathbb{C}$ s.t. normalised and positive $\Rightarrow$ GNS-theorem $(\mathscr{H}, \pi, \Omega)$
- **Locally covariant QFT**
  - ✓ $\mathfrak{A} : \mathsf{Loc} \to \mathsf{Alg}$
    - **Locality:** $f : \mathcal{M} \to \mathcal{M}' \Longrightarrow \mathfrak{f} : \mathcal{A}(\mathcal{M}) \to \mathcal{A}(\mathcal{M}')$ injective
    - **Causality:** $\mathcal{M}_1 \xrightarrow{f_1} \mathcal{M} \xleftarrow{f_2} \mathcal{M}_2$ caus. disjoint $\Longrightarrow [\mathfrak{f}_1\mathcal{A}(\mathcal{M}_1), \mathfrak{f}_2\mathcal{A}(\mathcal{M}_2)] = 0$
    - **Time-slice:** $f : \mathcal{M} \to \mathcal{M}'$ s.t. $f(\mathcal{M}) \supset \Sigma' \Longrightarrow \mathfrak{f}$ is isomorphism
  - ⚡ **natural state** $f : \mathcal{M} \to \mathcal{M}' \Longrightarrow \mathfrak{f}^*(\omega_{\mathcal{M}'}) = \omega_{\mathcal{M}}$

GOAL: **construct ω non-locally determined by the geometry**

Outline of the Talk

- **Algebraic approach to quantum Dirac fields**
- **Functional analytic constructions for quasifree states**

Based on:

- my PhD Thesis (soon available on the arXiv)
- and on previous papers with
  - F. Finster, C. Röken (arXiv:1501.05522 -JMP; arXiv:1606.03882)
  - N. Drago (arXiv:1607.02909)
  - C. Dappiaggi, H. Gimperlein, A. Schenkel, (arXiv:1512.07823)
  - M. Benini, C. Dappiaggi (arXiv:1404.4551 -JMP)

PART I:

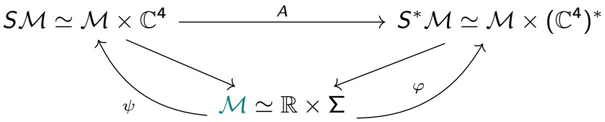
Algebraic approach to quantum Dirac fields

AQFT - I: Kinematics

- $\mathcal{M}$ is 4-dim **globally hyperbolic spacetime** :

$$ds^2 = \beta^2 dt^2 - h_t; \quad \beta \in C^\infty(M; \mathbb{R}^+) \text{ and } h_t \in \mathrm{Riem}(\Sigma); \forall t \in \mathbb{R}$$

- **Spinor bundle** $S\mathcal{M}$ and **cospinor bundle** $S^*\mathcal{M}$



- **Spin product** $\prec \cdot \,|\, \cdot \succ_x \, : \Gamma(S\mathcal{M}) \times \Gamma(S\mathcal{M}) \to \mathbb{C}$

$$\prec\psi|\widetilde{\psi}\succ_x := ((A\psi)\widetilde{\psi})(x).$$

AQFT - II: Dynamics

- **Dirac operator** on SM and its dual on S^*M:

$$\mathcal{D}\psi \doteq (i\gamma^\mu\nabla_\mu + \mathcal{B} - m)\psi = 0, \qquad \mathcal{D}^*\varphi = (-i\gamma^\mu\nabla_\mu + \mathcal{B} - m)\varphi = 0.$$

- **Causal propagators:** $G^{(*)} : \Gamma_c(S\mathcal{M}^{(*)}) \to \Gamma_{sc}(S\mathcal{M}^{(*)})$

$$\mathcal{D}^{(*)} \circ G^{(*)} = 0 = G^{(*)} \circ \mathcal{D}^{(*)}|_{\Gamma_c(S\mathcal{M}^{(*)})}$$
$$supp(G^{(*)}(f)) \subseteq J(supp(f))$$

- **Hilbert spaces:**

$$\mathscr{H}^s := \overline{\left(\mathcal{S}ol(\mathcal{D}) \simeq \frac{\Gamma_c(S\mathcal{M})}{\mathcal{D}\Gamma_c(S\mathcal{M})}, (\cdot \mid \cdot)^s \doteq \int_\Sigma \prec \cdot \,|\!\!\not{\psi}\, \cdot \succ_x d\Sigma\right)}$$

$$\mathscr{H}^c := \overline{\left(\mathcal{S}ol(\mathcal{D}^*) \simeq \frac{\Gamma_c(S\mathcal{M}^*)}{\mathcal{D}^*\Gamma_c(S\mathcal{M}^*)}, (\cdot \mid \cdot)^c \doteq \int_\Sigma \prec A^{-1} \cdot |\!\!\not{\psi}\, A^{-1} \cdot \succ_x d\Sigma\right)}$$

AQFT - III: CAR Algebra

- Set of generators:

$$1_{\mathcal{T}} = \{1, 0, \dots\}, \quad \Phi(\psi) = \left\{0, \begin{pmatrix}\psi \\ 0\end{pmatrix}, 0, \dots\right\} \quad , \Psi(\varphi) = \left\{0, \begin{pmatrix}0 \\ \varphi\end{pmatrix}, 0, \dots\right\}$$

- Universal tensor C^*-algebra: $\mathcal{T} = \bigoplus_{k=0}^{\infty} \left(\mathscr{H}^s \oplus \mathscr{H}^c\right)^{\otimes k}$

$$\left\{0, 0, \begin{pmatrix}\psi_m \\ \varphi_m\end{pmatrix} \otimes \begin{pmatrix}\widetilde{\psi}_m \\ \widetilde{\varphi}_m\end{pmatrix}, \dots\right\}^* = \left\{0, 0, \begin{pmatrix}A^{-1}\widetilde{\varphi}_m \\ A\widetilde{\psi}_m\end{pmatrix} \otimes \begin{pmatrix}A^{-1}\varphi_m \\ A\psi_m\end{pmatrix}, \dots\right\}$$

- We encode the **C**anonical **A**nticommutation **R**elations in the ideal $\mathcal{I} \subset \mathcal{T}$:
  - $\Phi(\psi) \otimes \Phi(\widetilde{\psi}) + \Phi(\widetilde{\psi}) \otimes \Phi(\psi)$
  - $\Psi(\varphi) \otimes \Psi(\widetilde{\varphi}) + \Psi(\widetilde{\varphi}) \otimes \Psi(\varphi)$
  - $\Psi(\varphi) \otimes \Phi(\psi) + \Phi(\psi) \otimes \Psi(\varphi) - \left(A^{-1}\varphi \mid \psi\right)^s 1_{\mathcal{T}}$

- **Algebra of Dirac fields:** $\mathcal{F} \doteq \dfrac{\mathcal{T}}{\mathcal{I}}$

AQFT - IV: States

- **Algebraic state** $\omega : \mathcal{F} \to \mathbb{C} \quad \omega(1_{\mathcal{F}}) = 1, \quad \omega(h^* h) \geq 0, \quad \forall\, h \in \mathcal{F}.$

N.B.: Choosing a state ω is equivalent to assigning $\omega_n(h_1, \ldots, h_n)$ $\forall n \in \mathbb{N}$ and $\forall h_i \in \mathcal{F}$.

- **Quasifree states:** $\omega_{2n+1}(h_1, \ldots, h_{2n+1}) = 0$

$$\omega_{2n}(h_1, \ldots, h_{2n}) = \sum_{\sigma \in S'_{2n}} (-1)^{\mathrm{sign}(\sigma)} \prod_{i=1}^{n} \omega_2\left(h_{\sigma(2i-1)}, h_{\sigma(2i)}\right).$$

Question: **Are all states physically acceptable?**

Of course not! Minimal requirements are:

i) covariant construction of Wick polynomials to deal with interactions,

ii) same UV behaviour of the Minkowski vacuum,

iii) finite quantum fluctuations of all observables.

Answer: **Hadamard States**

AQFT - V: Hadamard States

- A (quasifree) state ω satisfies the **Hadamard condition** if and only if

$$WF(\omega_2) = \{(x, y, \xi_x, \xi_y) \in T^*M^{\otimes 2} \setminus 0 \mid (x, \xi_x) \sim (y, -\xi_y), \quad \xi_x \triangleright 0\}$$

where $(x, \xi_x) \sim (y, -\xi_y)$ means that x and y are connected by a null geodesic and $-\xi_y$ is the parallel transport of the co-parallel co-vector ξ_x; whereas $\xi_x \triangleright 0$ implies that ξ_x is future-pointing.

Question: **How many Hadamard states do we know?**

- deformation arguments (existence)
- static spacetime (H. Sahlmann and R. Verch)
- highly symmetric spacetimes, e.g., de Sitter spacetime
- holographic techniques (C. Dappiaggi, V. Moretti and N. Pinamonti)
- pseudodifferential calculus (C. Gérard and M. Wrochna)

Question: **Does there exist an explicit method that does not use the symmetries?**

PART II:

Functional analytic constructions for quasifree states

Characterisation of quasifree states

Lemma 1 *(H. Araki: On quasifree states of CAR and Bogoliubov automorphisms.)*

Let Υ be an involution on $\mathscr{H} := \mathscr{H}^s \oplus \mathscr{H}^c$ and $Q := Q^s \oplus Q^c$ be a *bounded, symmetric operator* on $\mathscr{H}$ with the following properties

(a) $Q + \Upsilon Q \Upsilon = Id$,

(b) $0 \leq Q = Q^* \leq 1$.

Then there exists a unique quasifree state ω on $\mathcal{F}$ such that

$$\omega_2\Big(\Psi(\varphi)\Phi(\psi)\Big) = \left(A^{-1}\varphi \mid Q^s\psi\right)^s$$

Lemma 2 *(N. Drago and S. M.: arXiv:1607.02909)*

Let Υ be an involution on $\mathscr{H}$ and Π be a orthonormal projector on $\mathscr{H}^s$. Then

$$P := \Pi \oplus (Id - A\Pi A^{-1})$$

satisfies (a) and (b).

Quantum states in Rindler spacetime - I

[F. Finster, S. M. and C. R oken: The fermionic signature operator and quantum states in Rindler space-time, arXiv:1606.03882.]

$$\mathcal{R} = \{(t,x) \in \mathbb{R}^{1,1} \quad \text{with} \quad |t| < x\} \text{ with element line } ds^2 = dt^2 - dx^2$$

- Rindler coordinates:

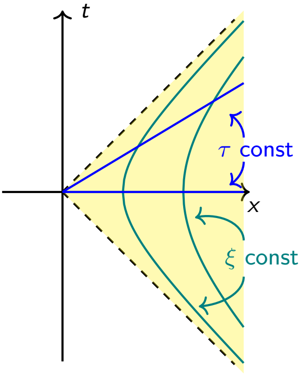
$$\begin{cases} t = \xi \sinh \tau \\ x = \xi \cosh \tau \end{cases} \quad \Rightarrow \quad ds^2 = \xi^2 d\tau^2 - d\xi^2$$

- γ-matrices: $\{\gamma^a, \gamma^b\} = 2g^{ab}\, 1_{\mathbb{C}^2}$

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- Dirac operator:

$$\mathcal{D} = i\gamma^0\partial_t + i\gamma^1\partial_x - 1_{\mathbb{C}^2}\, m$$



Quantum states in Rindler spacetime - II

Products and pairing

$$\psi \in \Gamma_{sc}(S\mathcal{R}) \text{ and } S\mathcal{R} = \mathcal{R} \times \mathbb{C}^2 \subset \mathcal{M} \times \mathbb{C}^2 = S\mathcal{M} \text{ and } \Psi \in \Gamma_{sc}(S\mathcal{M})$$

- **spin product**: $\prec \cdot \mid \cdot \succ = \langle \cdot , \gamma^0 \cdot \rangle_{\mathbb{C}^2}$
- **scalar products**:

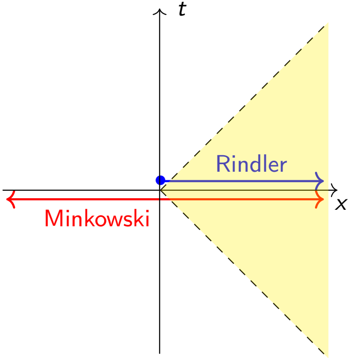
$$(\cdot \mid \cdot)_{\mathcal{R}} := 2\pi \int_0^\infty \prec \cdot \mid \gamma^0 \cdot \succ |_{(t=0,x)}\, dx$$

$$(\cdot \mid \cdot)_{\mathcal{M}} := 2\pi \int_{-\infty}^\infty \prec \cdot \mid \gamma^0 \cdot \succ |_{(t=0,x)}\, dx$$

- **spacetime pairing**:

$$\langle \cdot \mid \cdot \rangle_{\mathcal{R}} := 2\pi \int_{\mathcal{R}} \prec \cdot \mid \cdot \succ \; dt\, dx$$

$$\langle \cdot \mid \cdot \rangle_{\mathcal{M}} := 2\pi \int_{\mathcal{M}} \prec \cdot \mid \cdot \succ \; dt\, dx$$



Quantum states in Rindler spacetime - III

Embedding in Minkowski

Lemma 3

$$\mathscr{H}_{\mathcal{R}} := \overline{(\mathcal{S}ol(\mathcal{R}), (\cdot \mid \cdot)_{\mathcal{R}})} \text{ and } \mathscr{H}_{\mathcal{M}} = \overline{(\mathcal{S}ol(\mathcal{M}), (\cdot \mid \cdot)_{\mathcal{M}}\, dx)}$$

- Then there exist

  (i) $\iota_{\mathcal{M}} : \mathscr{H}_{\mathcal{R}} \to \mathscr{H}_{\mathcal{M}}$

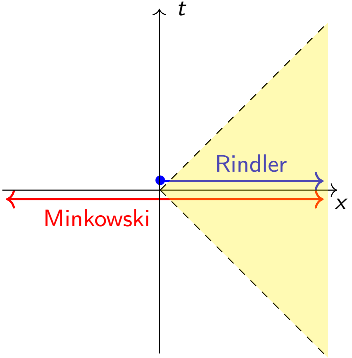
  (ii) $\pi_{\mathcal{R}} : \mathscr{H}_{\mathcal{M}} \to \mathscr{H}_{\mathcal{R}}$

$$\iota_{\mathcal{M}} \circ \pi_{\mathcal{R}} = Id_{\mathscr{H}}$$

- Moreover, $\forall\ \Psi \in \mathscr{H}_{\mathcal{M}}$ and $\psi \in \mathscr{H}_{\mathcal{R}}$,

$$(\Psi \mid \iota_{\mathcal{M}}\psi)_{\mathcal{M}} =$$
$$= 2\pi \int_0^\infty \prec \Psi \mid \gamma^0 \psi \succ |_{(0,x)}\, dx =$$
$$= (\pi_{\mathcal{R}}\Psi \mid \psi)_{\mathcal{R}} ,$$

  which can be written as $\iota_{\mathcal{M}}{}^* = \pi_{\mathcal{R}}$.



Quantum states in Rindler spacetime - IV

Fermionic Signature Operators

$$\lightning\ \langle\cdot\mid\cdot\rangle_{\mathcal{M}} : \mathcal{S}ol(\mathcal{M})\times\mathcal{S}ol(\mathcal{M})\to\mathbb{C} \implies \langle\cdot\mid\cdot\rangle_{\mathrm{Rel}} := \int_{\mathcal{M}}\chi_{\mathcal{R}} \prec\cdot\mid\cdot\succ\, dt\, dx$$

Lemma 4

- $\forall\tilde{\Psi}\in\mathscr{H}_{\mathcal{M}}$, $\exists c=c(\tilde{\Psi})$ s.t. $\left|\left\langle\Psi\mid\tilde{\Psi}\right\rangle_{\mathrm{Rel}}\right|\leq c(\tilde{\Psi})\,\|\Psi\|_{\mathcal{M}}$ for every $\Psi\in\mathscr{H}_{\mathcal{M}}$.

Definition 1

- **Relative fermionic signature operator** $\mathcal{S}_{\mathrm{Rel}} : D(\mathcal{S}_{\mathrm{Rel}})\subset\mathscr{H}_{\mathcal{M}}\to\mathscr{H}_{\mathcal{M}}$

$$\left\langle\Psi\mid\tilde{\Psi}\right\rangle_{\mathrm{Rel}} = \left(\Psi\mid\mathcal{S}_{\mathrm{Rel}}\tilde{\Psi}\right)_{\mathcal{M}}\qquad\forall\Psi\in\mathscr{H}_{\mathcal{M}}\,.$$

- **Fermionic signature operator** $\mathcal{S} : D(\mathcal{S})\subset\mathscr{H}_{\mathcal{R}}\to\mathscr{H}_{\mathcal{R}}$

$$\mathcal{S}=\pi_{\mathcal{R}}\,\mathcal{S}_{\mathrm{Rel}}\,\iota_{\mathcal{M}}\qquad\text{with}\qquad D(\mathcal{S})=\pi_{\mathcal{R}}\big(D(\mathcal{S}_{\mathrm{Rel}})\big)\,.$$

N.B.: $\mathcal{S}_{\mathrm{Rel}}$ and $\mathcal{S}$ are densely defined, symmetric and unbounded.

Quantum states in Rindler spacetime - V

Transformation to the Momentum Space

$$\psi(x,t) = \frac{1}{4\pi^2}\int \widehat{\psi}(\omega,k)d\omega dk = \frac{1}{4\pi^2}\int \widehat{\psi}(s,\alpha)d\alpha$$

- **in the parametrization** $\begin{pmatrix}\omega \\ k\end{pmatrix} = ms\begin{pmatrix}\cosh\alpha \\ \sinh\alpha\end{pmatrix}$ with $s\in\{\pm 1\}$ and $\alpha\in\mathbb{R}$,
- **scalar product** $\left(\Psi \mid \tilde{\Psi}\right)_{\mathcal{M}} = \frac{1}{4m}\sum_{s=\pm 1}\int_{-\infty}^{\infty}\overline{g(s,\alpha)}\,\tilde{g}(s,\alpha)\,d\alpha$
- **spacetime pairing**

$$\left\langle \Psi \mid \tilde{\Psi}\right\rangle_{\mathrm{Rel}} = \frac{1}{4m}\sum_{s=\pm 1}\int_{-\infty}^{\infty}d\alpha\,\overline{g(s,\alpha)}\,\overbrace{\sum_{\tilde{s}=\pm 1}\lim_{\varepsilon\searrow 0}\int_{-\infty}^{\infty} I_\varepsilon\big(s,\alpha;\tilde{s},\tilde{\alpha}\big)\;\tilde{g}(\tilde{s},\tilde{\alpha})\,d\tilde{\alpha}}^{\widehat{\mathcal{S}}_{\mathrm{Rel}}\widehat{\tilde{\Psi}}(s,\alpha)\equiv}$$

where I_ε is the kernel

$$I_\varepsilon\big(s,\alpha;\tilde{s},\tilde{\alpha}\big) = \frac{1}{4\pi^2\,m}\times\begin{cases}\dfrac{s\cosh\left(\frac{\alpha-\tilde{\alpha}}{2}\right)}{1-\cosh(\alpha-\tilde{\alpha}+i\varepsilon s)} & \text{if } s=\tilde{s}\\[2ex] -\dfrac{s\sinh\left(\frac{\alpha-\tilde{\alpha}}{2}\right)}{1+\cosh(\alpha-\tilde{\alpha})} & \text{if } s\neq\tilde{s}\end{cases}$$

Quantum states in Rindler spacetime - VI

The Self-Adjoint Extension

- Unitary operator

$$U : \mathscr{H}_{\mathcal{M}} \to L^2(\mathbb{R}, \mathbb{C}^2)\,, \qquad g(s,\alpha) \mapsto \hat{g}(s,\ell) = \frac{1}{\sqrt{8\pi m}} \int_{-\infty}^{\infty} g(s,\alpha)\, e^{i\ell\alpha}\, d\alpha$$

- **Relative fermionic signature operator** $\big(\mathcal{S}_{\mathrm{Rel}}\Psi\big)(s,\alpha) = \big(U^{-1}\,\hat{\mathcal{S}}_{\mathrm{Rel}}\,U\Psi\big)(s,\alpha)$

$$\hat{\mathcal{S}}_{\mathrm{Rel}}(\ell) = \frac{\ell}{\pi m} \begin{pmatrix} \dfrac{1}{1+e^{-2\pi\ell}} & -\dfrac{i}{2\cosh(\pi\ell)} \\ \dfrac{i}{2\cosh(\pi\ell)} & \dfrac{1}{1+e^{2\pi\ell}} \end{pmatrix}$$

Lemma 5

- $\mathcal{S}_{\mathrm{Rel}}$ has **unique self adjoint extension** with

$$D(\mathcal{S}_{\mathrm{Rel}}) = U^{-1}\Big(\Big\{\hat{g} \in L^2(\mathbb{R},\mathbb{C}^2) \text{ with } \hat{\mathcal{S}}_{\mathrm{Rel}}\,\hat{g} = \in L^2(\mathbb{R},\mathbb{C}^2)\Big\}\Big) \text{ where}$$

- **Spectrum:** $\sigma_{\mathrm{pp}}\big(\mathcal{S}_{\mathrm{Rel}}\big) = \{0\}$ and $\sigma_{\mathrm{ac}}\big(\mathcal{S}_{\mathrm{Rel}}\big) = \mathbb{R}$
- **Spectral measure** dE_λ is given by $E_I = U^{-1}\Big(\chi_I(0)\,\hat{K} + \chi_I\,\frac{\pi m}{\ell}\,\hat{\mathcal{S}}_{\mathrm{Rel}}(\ell)\Big)U$

$$\hat{L}(\ell) = \frac{\pi m}{\ell}\,\hat{\mathcal{S}}_{\mathrm{Rel}}(\ell) \qquad \hat{K}(\ell) = 1_{\mathbb{C}^2} - \hat{L}(\ell)$$

Quantum states in Rindler spacetime - VII

Connection with the Hamiltonian and Quasifree States

Lemma 6

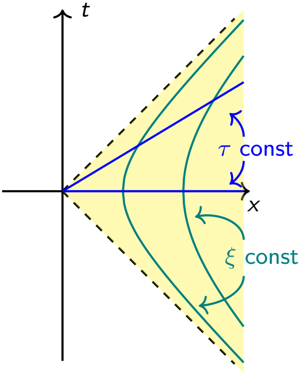
- $\mathcal{S}$ has unique **self-adjoint** extension in the domain $D(\mathcal{S}) = \pi_{\mathcal{R}} D(\mathcal{S}_{\text{Rel}})$
- **Spectral measure** is given by $E_I = \pi_{\mathcal{R}}\, U^{-1}\big(\chi_I(\frac{\pi m}{\ell}\,\hat{\mathcal{S}}_{\text{Rel}})\big)\, U\, \iota_{\mathcal{M}}$
- **Spetrum**: $\sigma_{\text{ac}}(\mathcal{S}) = \mathbb{R}$

- Change of coordinates $(t, x) \mapsto (\tau, \xi)$
- Dirac equation in the **Hamiltonian** form

$$i\partial_\tau \psi = \mathbf{H}\psi$$

Lemma 7

- **Fermionic signature operator** $\quad \mathcal{S} = -\dfrac{\mathbf{H}}{\pi m}$
- **Ground state:** $\quad \chi(\mathcal{S}) = \chi(\mathbf{H})$



Quantum states in Rindler spacetime - VIII

Extension to the Four Dimensional Rindler Spacetime

$$\mathcal{R} = \left\{(t,x,y,z) \in \mathbb{R}^{1,3} \quad \text{with} \quad |t| < x\right\} \text{ with } ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

- **Solutions:** $\psi(t,x,y,z) = e^{ik_y y + ik_z z}\,\tilde{\psi}(t,x)$
- **Massive Dirac equation:** $\left(i\gamma^0\partial_t + i\gamma^1\partial_x\right)\tilde{\psi}(t,x) = \left(m + \gamma^2 k_y + \gamma^3 k_z\right)\tilde{\psi}(t,x)$

Lemma 8

- The fermionic signature operator $\mathcal{S}$ and the Hamiltonian H satisfy

$$\mathcal{S} = -\frac{\mathrm{H}}{\pi\,\tilde{m}} - \frac{1}{2\pi\, m\tilde{m}}\,\gamma^0\gamma^1\left(\gamma^2\partial_y + \gamma^3\partial_z\right)$$

  being $\tilde{m} := \sqrt{m^2 + k_y^2 + k_z^2}$.

- **Quasifree state:** $\chi(\mathcal{S}) \neq \chi(\mathbf{H}) \quad \Longrightarrow$ ¿¿ physical interpretation ??

Conclusions

What we know:

- Every orthonormal projector implicitly defines a quasifree state.
- The construction gives an Hadamard state in 1+1 Rindler spacetime.

Benefit:

- This technique works without symmetries.

Flaws:

- This technique works only for massive fields.

Future investigations:

- How can we extend this technique to more general spacetime?
- Does the state in 4-dim satisfy the Hadamard condition?